RADIO SCIENCE, VOL. 42, RS2S02, doi:10.1029/2006RS003495, 2007

Synopsis of the historical development of Schumann resonances

B. P. Besser[1]

Received 7 March 2006; revised 30 November 2006; accepted 8 December 2006; published 28 March 2007.

[1] The life and work of Winfried Otto Schumann and the historical development of the main ideas leading to the hypothesis of Earth-ionosphere cavity electromagnetic oscillations are reviewed. The so-called Schumann resonances are a set of frequencies of electromagnetic waves in the natural cavity formed by a planet's (moon's) surface and its ionosphere, in the extremely low frequency (ELF) range, caused by natural electrical activity of the planet (moon) and/or its atmospheric environment. Additionally, the reception of his work by the contemporary scientific community as well as the experimental evidence for the postulated ELF resonance oscillations is described.

Citation: Besser, B. P. (2007), Synopsis of the historical development of Schumann resonances, *Radio Sci.*, *42*, RS2S02, doi:10.1029/2006RS003495.

1. Introduction

[2] In 1952, Winfried Otto Schumann, a professor at the Technische Hochschule München, Germany, published several papers postulating the resonances of extremely low frequency (ELF) waves in the Earth-air-ionosphere waveguide excited by lightning discharges. His idea was composed of three topics: (1) the propagation of electromagnetic waves in a spherical cavity, (2) the Earth-air-ionosphere system acting as a waveguide, and (3) excitation by lightning discharges. This report tries to falsify for at least one case the so-called "zeroth theorem of science history," a saying (one-liner) among science historians, which claims that a discovery (rule, insight, etc.) named after a person was not (necessarily) inaugurated by this person [*Fischer*, 2006].

1.1. Literature Search

[3] During the last four years a rather extensive literature search had been undertaken to unveil some historical facts related to the physics of the so-called "Schumann resonances." Although the search led to a vast amount of literature on the topic starting in the middle of the 1960s, references predating Schumann's publications were rather scarce. Nevertheless, an even more profound search in wave propagation-related journals and books unearthed some interesting pieces of real and pretended precursors to Winfried Schumann.

[4] Although extensive, any literature search can never be entirely exhaustive, and in particular, the one performed for this study was limited mainly with respect to the Russian literature, and therefore it cannot be ruled out that this synopsis misses some relevant pieces. If this is the case, the author would be deeply grateful to get notice from the reader.

1.2. Additional Reading

[5] The scope of this report is limited to prime sources of the pre-Schumann era (before 1952) and to the main developments in the field during roughly the first decade after Schumann predicted the Earth-ionosphere cavity oscillations. Additionally, this report is not supposed to render earlier reviews on this physical topic obsolete; it should rather complement them. The books by *Budden* [1961], *Wait* [1962], *Blackband* [1964], *Volland* [1968], *Watt et al.* [1967], *Galejs* [1972], *Burrows* [1978], *Bliokh et al.* [1980], and *Nickolaenko and Hayakawa* [2002] and the reviews by *Polk* [1982], *Sentman* [1995], and *Barr et al.* [2000] are recommended to find out more on the physics and mathematical treatment of the phenomena, for which we can only scratch the surface in this review. For lightning-related topics the book by *Rakov and Uman* [2005] is recommended as recent authoritative work.

[6] Any paper on ELF (or VLF) wave propagation, and certainly a historical review, should mention the "monumental" work of James R. Wait (1924–1996), who published in sum over 800 papers and eight books on electromagnetics. He was an outstanding theoretician

---

[1] Space Research Institute, Austrian Academy of Sciences, Graz, Austria.

Copyright 2007 by the American Geophysical Union.
0048-6604/07/2006RS003495

Figure 1. Winfried Otto Schumann on a United States Marine troop carrier during the Atlantic crossing to the United States, 1947 (information from Wolfgang Schumann, Munich, courtesy of Historisches Archiv, Technische Universität München, Munich, Germany).

and contributed to all the different aspects of very low frequency (VLF) and ELF wave propagation. A full appreciation of Wait's work is outside the scope of this paper, and therefore the reader is referred to some recollections of Wait's colleagues, e.g., the paper by *Smith* [2000, and references therein].

1.3. Outline

[7] Schumann published only in German, and up to now only scarce information about his life and work has been available in the English scientific/technical literature. Therefore a short biographical summary will be given in section 2. In sections 3 and 4 the historical development of the problem of propagation of electromagnetic waves in a cavity between conducting spherical shells and the waveguide concept of the Earth-ionosphere system with examples from the scientific/technical literature will be outlined. A discussion about Schumann's work regarding the long wavelength oscillations in the Earth-ionosphere cavity will be carried out in section 5, followed by the final sections devoted to the reception of Schumann's work by his contemporaries and the early observational evidence for the existence of "Schumann resonances."

2. Winfried Otto Schumann (1888–1974)

2.1. Biographical Sketch

[8] Winfried Otto Schumann was born on 20 May 1888 in Tübingen, Germany, the son of the physical chemist Ernst Otto Schumann (1852–1898) [*Poggendorff*, 1904]. Because of his father's several job-related relocations, he grew up in different places in German-speaking countries, among them Berndorf, southeast of Vienna (Austria), and Karolinental, near Prague (now Karlín, part of Praha, the capital of the Czech Republic).

[9] From 1905 to 1909 he studied electrical engineering at the Technische Hochschule Karlsruhe (now named Universität Karlsruhe, the first German polytechnic, nowadays equivalent to a technical university) and worked subsequently as assistant to the founder of its Institute of Electrical Engineering, Engelbert Arnold (1856–1911) [*Poggendorff*, 1956; *Killy*, 1995]. During this time, Schumann prepared his doctoral thesis "On the torques of the damper winding of a multiphase synchronous machine at small pendulum oscillations in parallel operation" under Arnold's guidance (until his death in November 1911) and earned his doctorate degree "Dr.-Ing." after his viva voce in 1912.

[10] After his final examination he started to work in industry as head of the High-Voltage Laboratory for the company Brown, Boveri & Cie at Baden, Switzerland, until 1914. During the First World War he served as a radio operator, and beginning in 1919 he worked as research assistant of the Robert-Bosch-Stiftung (Robert Bosch Foundation) in the Institute of Electrical Engineering at the Technische Hochschule Stuttgart (now Universität Stuttgart). There he also qualified for university teaching ("Habilitation") in 1920 with a thesis on "Electrical breakdown stress of gases." In the same year he was appointed Associate Professor ("Außerordentlicher Universitätsprofessor") of Technical Physics at the University in Jena, Germany.

[11] In 1924 he was appointed full professor ("Ordentlicher Universitätsprofessor") for theoretical electrical engineering at the newly founded Electro-Physical Laboratory at the Technische Hochschule München (since 1970 named Technische Universität München), Germany. This laboratory was later upgraded and renamed Institute of Electrophysics. From September 1947 to October 1948 Schumann was on leave and worked at Wright Airfield (later renamed Wright-Patterson Air Force Base), Dayton, Ohio, for the United States Air Force (see Figure 1). In 1961 he was given the status of professor emeritus, but he remained active in research until his death on 24 September 1974.

2.2. Schumann's Scientific Activities

[12] Four distinct periods can be distinguished in the scientific career of Winfried Otto Schumann. The first started with his dissertation and ended with his appointment as full professor in Munich and was mainly dedicated to topics in high-voltage engineering. From the beginning of his professorship in Munich, 1924, until 1951 (the second period) he worked on discharge phenomena in highly ionized gases (plasmas) and also on wave propagation therein. The third period, from 1952 to about 1957, was devoted to investigations of the propagation of ELF waves in the cavity between the Earth's surface and the lower ionosphere. After 1958 and extending after his retirement he worked mainly on problems of the motion of electrical charges under the influence of low-frequency electromagnetic fields. Additionally, he served for several years as a member of the steering committee of the Deutsches Museum in Munich, one of the world's largest museums for natural sciences and technology, and as a member of the administrative board of the Bavarian Broadcasting Service (Bayerischer Rundfunk).

[13] His scientific work was honored in 1945 when he was named a member of the Bayerische Akademie der Wissenschaften (Bavarian Academy of Sciences and Humanities) in 1957 by the granting of an honorary doctorate by the Technische Hochschule Darmstadt and in 1963 when he was awarded the Großes Verdienstkreuz der Bundesrepublik (Commander's Cross of the Order of Merit of the Federal Republic of Germany).

[14] For further details of his scientific career, the reader is referred to short entries in bibliographical/biographical dictionaries [*Poggendorff*, 1940; *Poggendorff*, 1961; *Killy and Vierhaus*, 1998], the main biographical sketches in the technical literature [*Medicus*, 1948; *Piloty*, 1958; *Knoll*, 1968], and the obituary by Gottfried Eckart (1906–1999), a former colleague of earlier Munich times and since 1954 professor at the University of Saarbrücken [*Eckart*, 1975]. The paper by his long-time professorial colleague Hans Piloty (1894–1969) dedicated to Schumann's 70th birthday also includes a comprehensive (but not complete) listing of 68 publications [*Piloty*, 1958].

[15] All these biographies are published in German, and the only biographical sketch published in English focuses on Schumann's contributions to the fields of discharge phenomena and insulating materials [*Stäblein and Heilbronner*, 1991]. For the 50th anniversary of the postulation of the Schumann resonances, a commemorative article was published in a German physics magazine [*Schlegel and Füllekrug*, 2002]. Some additional material about Schumann and his professional activities in Munich is available at the Historisches Archiv der Technischen Univerität München (Historical Archive of the Technische Universität München).

3. Electromagnetic Oscillations Between Conducting Concentric Spheres

[16] This section gives some examples of investigations of the problem of electromagnetic oscillations (wave propagation) in the space between concentric spheres in the pre-Schumann era (years before 1952). One has to emphasize that the calculation of these oscillations with this simple model can only serve as a crude approximation to reality. First, neither the lower boundary (Earth) nor the upper boundary (lower boundary of the ionosphere) are perfect conductors; second, the electrical conductivity of the air is an increasing function with height; and third, the Earth's magnetic field and its effects on the propagation are not taken into account at all. As a consequence, the results of the calculations of a "perfect cavity" differ in several aspects from the ones which model the Earth-ionosphere waveguide more realistically (and in general cannot be solved by simple analytical methods). In a lossless system the resonance frequencies are lines in frequency space. However, losses in the medium of the real Earth-ionosphere cavity lead to a broadening of the resonance lines and lowering of the resonance peak frequencies in comparison with the "ideal case." The quality factors for the Earth-ionosphere cavity are in the range between 4 and 6 (depending on the mode) and are very small in comparison to technical systems (waveguides or microwave cavities), where the losses are mainly concentrated in the walls of the system and not in the medium.

3.1. Joseph J. Thomson's Investigations

[17] The first mention of electromagnetic oscillations in a cavity between concentric spheres can be found in the book on electricity and magnetism by Joseph J. Thomson (1856–1940) published in 1893 [*Thomson*, 1893]. Thomson became famous for demonstrating the existence of the electron and measuring its charge-to-mass ratio in 1897. He won the Nobel Prize in Physics in 1906 for his "Investigations on passage of electricity through gases." For a short biography and a review of Thomson's scientific work the reader is referred to *Heilbron* [1976].

[18] In his book of 1893, he discussed electrical oscillations in a spherical conductor and gave due credit to the work of several predecessors, including George G. Stokes (1819–1903), John W. Strutt (also known as Lord Rayleigh) (1842–1919), Charles Niven (1845–1923), Horace Lamb (1849–1934), Hermann von Helmholtz (1821–1894), and Heinrich (Eduard) Heine (1812–1881).

[19] In a chapter "Electrical waves and oscillation," on the oscillations between two concentric spherical conductors, Thomson wrote [*Thomson*, 1893, pp. 372–376]:

> 315.] To consider more closely the effect of reflection let us take the case of two concentric spherical conductors of radius **a** and **b** respectively.
>
> We may show, as in Art. 311 [equation giving the period of vibrations (comment added by the present author)], that if the spheres are metallic and not excessively small the electromotive intensity parallel to the surface of the spheres vanishes when $r = \mathbf{a}$ and when $r = \mathbf{b}$; . . .
>
> This case is that of the vibration of a spherical shell excited by some cause inside, here there is no radiation of the energy into space, the electrical waves keep passing backwards and forwards from one part of the surface of the sphere to another. . ..
>
> Electrical Oscillations on Two Concentric Spheres of nearly equal radius.
>
> 316.] When d, the difference between the radii **a** and **b**, is very small compared with **a** or **b**, . . .
>
> There will be one root of this equation corresponding to a vibration whose wavelength is comparable with **a**, and other roots corresponding to wavelengths comparable with d. . . .The wavelength is thus equal to $\pi\sqrt{2}$ times the radius of the sphere.

This expression for the wavelength of the lowest resonance frequency wave is the same as the one Schumann derived in 1952 for the analogous Earth-ionosphere system.

3.2. George F. FitzGerald's Contribution

[20] Tracing a passing mention in the introduction of a paper by Frederick W. Chapman (1906–1985) and David Llanwyn Jones (born 1938) [*Chapman and Jones*, 1964] from the Wheatstone Laboratory of King's College, London, we learn that the Irish physicist George F. FitzGerald (1851–1901) in the middle of September 1893 presented the following findings at a talk during the 63rd Meeting of the British Association of the Advancement of Science in Nottingham, United Kingdom [*Nature*, 1893, p. 526; *FitzGerald*, 1902, pp. 301–302]:

> Prof. G. F. Fitzgerald gave an interesting communication on "The period of vibration of Disturbances of Electrification of the earth". The period of oscillation of a simple sphere of the size of the earth, supposed charged with opposite charges of electricity at its ends, would be about 1/17th of a second; but the hypothesis that the earth is a conducting body surrounded by a non-conductor, is not in accordance with the fact. Probably the upper regions of our atmosphere are fairly good conductors. In a Geissler tube air is a good conductor, and we know that when part of a gas is transmitting an electrical disturbance the rest of the gas in its neighborhood becomes capable of transmitting such as well. Extending the analogy, we may assume that during a thunderstorm the air becomes capable of transmitting small disturbances. If the earth is surrounded by a conducting shell its capacity may be regarded as that of two concentric spheres, and is accordingly greater than that of a simple sphere, which would produce a corresponding change in the oscillation. If we assume that the height of the region of the aurora to be 60 miles or 100 kilometers, we get a period of oscillation of 0.1 second. Assuming it to be 6 miles (or 10 km.) the period becomes 0.3 seconds. . ..

[21] Digging out FitzGerald's original (rather) short contribution in the proceedings of the conference reveals the following [*FitzGerald*, 1893]:

> Professor J.J. Thomson and Mr. O. Heaviside have calculated the period of vibration on a sphere alone in space and found it about 0.059 second. The fact that the upper regions of the atmosphere conduct makes it possible that there is a period of vibration due to the vibrations similar to those on a sphere surrounded by a concentric spherical shell. . .. The value of the time of vibration obtained by this very simple approximation is
>
> $$T = \pi\sqrt{\frac{2K\mu a^2 b^2 \log a/b}{a^2 - b^2}}$$
>
> Applying this to the case of the earth with a conducting layer at a height of 100 km (much higher than probable) it appears that a period of vibration of about one [sic (note added by the present author)] second would be possible. A variation in the height of the conducting layer produces only a small effect upon this if the height be small compared with the diameter of the earth. . ..

[22] In the above equation K and μ are the material permittivity and permeability, and a and b are the radii of the two spheres, correspondingly.

[23] Because of the fact that FitzGerald did not show the derivation of the formula for the period, one cannot infer where he made the "error" corresponding to the oscillation period being 1 s as opposed to the correct value of 0.1 s mentioned in the account published in *Nature* [*Nature*, 1893; *FitzGerald*, 1902].

[24] Nevertheless, it seems that FitzGerald was really the first to consider the so-called Schumann resonance problem in the frame of the then only vaguely suspected Earth-ionosphere concept. This hypothesis is also endorsed in a recent notice by *Jackson* [2003]; for details see section 3.4.

3.3. Joseph Larmor's Calculations

[25] Only one year after Thomson and FitzGerald investigated the aforementioned problem, Joseph Larmor (1857–1942), then lecturer at St John's College, Cambridge, United Kingdom, also calculated the free periods in a uniform spherical condenser in 1894 (assuming that the separation distance between the spheres is small in comparison to the inner sphere radius) and derived the rule for the periods [*Larmor*, 1929] (adapted by the present author):

$$\frac{2\pi a}{c}\,\frac{1}{\sqrt{n\cdot(n+1)}},$$

where a is the radius of the inner sphere, c is the speed of light, and n is an integer number. This formula is exactly the same one Schumann derived in 1952 and applied to the Earth-ionosphere cavity.

[26] For other contributions by Joseph Larmor to ionospheric physics in the early 1920s, the reader is referred

to a publication by another outstanding ionospheric physicist, Edward V. Appleton (1892–1965) [*Appleton*, 1961].

3.4. Nikola Tesla's Ideas

[27] Some postings on the Internet and even some Schumann resonance–related articles in the scientific literature are still spreading the rumor that the Schumann resonances had been originally suggested by the famous inventor and experimenter Nikola Tesla. Unfortunately, almost none of these publications gives any citation to a relevant Tesla publication for their claim, and the legend of Nikola Tesla being the "true prophet" of the Schumann resonances still persists.

[28] Nikola Tesla (1856–1943), a Serb, got his electrical engineering education at the Joanneum (Institute of Technology) in Graz (now Graz University of Technology), Austria. In 1884 he emigrated to the United States and worked initially for Thomas A. Edison (1847–1931) and later for George Westinghouse (1846–1914) and contributed extremely successfully to several electrical engineering fields, including communication.

[29] In one of Tesla's patents, U.S. Patent 787.412 (18 April 1905), one can find the following description [*Tesla*, 1905; p. 3, lines 19–62]:

> For the present it will be sufficient to state that the planet behaves like a perfectly smooth or polished conductor of inappreciable resistance with capacity and self induction uniformly distributed along the axis of symmetry of wave propagation and transmitting slow electrical oscillations without sensible distortion and attenuation.
>
> Besides the above three requirements seem to be essential to the establishment of the resonating condition.
>
> First. The earth's diameter passing through the pole should be an odd multiple of the quarter wavelength—that is, of the ratio between the velocity of light—and four times the frequency of currents.
>
> Second. It is necessary to employ oscillations in which the rate of radiation of energy into space in the form of hertzian or electromagnetic waves is very small. To give an idea, I would say that the frequency should be smaller than twenty thousand per second, though shorter waves might be practicable. The lowest frequency would appear to be six per second, in which case there will be but one node, at or near the ground-plate, and, paradoxical as it may seem, the effect will increase with the distance and will be greatest in a region diametrically opposite the transmitter. With oscillations still slower the earth, strictly speaking, will not resonate, but simply act as a capacity, and the variation of potential will be more or less uniform over its entire surface.
>
> Third. The most essential requirement is, however, that irrespective of frequency the wave of wave train should continue for a certain interval of time, which I have estimated to be not less than one-twelfth or probably 0.08484 of a second and which is taken in passing to and returning from the region diametrically opposite the pole over the earth's surface with a mean velocity of about four hundred and seventy-one thousand two hundred and forty kilometers per second. . ..

[30] In the introduction to a special issue of a scientific journal on ELF communication, James R. Wait cited some sentences from the above-quoted Tesla patent and was rather sympathetic with Tesla's ideas when he wrote [*Wait*, 1974a, p. 353, left column]:

> Nevertheless, many of Tesla's early experiments have an intriguing similarity with later developments in ELF communications. Tesla proposed that the earth itself could be set into a resonant mode at frequencies of the order of 10 Hz. He suggested that energy was reflected at the antipode of his Colorado Springs transmitter in such a manner that standing waves were set up. . ..

[31] Three years later in another paper [*Wait*, 1977], Wait had apparently dug further into Tesla's ideas and found the flaw hidden in Tesla's mechanical analogy for wireless telegraphy he had used in his patent, and Wait came to the following conclusion [*Wait*, 1977, p. 162, left column]:

> In fact, Tesla was convinced that the earth itself could be resonated by choosing a frequency that would permit constructive interference from a signal reflected at the antipode after having transversed a two-way path [via the center of the Earth! (comment by present author)]. . ..

[32] One of the exceptions also worth mentioning—who also got the appreciation for Tesla's idea right—is J. David Jackson (born 1925), professor emeritus of physics, University of California, Berkeley, in his book *Classical Electrodynamics* [*Jackson*, 1998], where he writes in a footnote in the section on "Earth and ionosphere as a resonant cavity: Schumann resonances" [*Jackson*, 1998, p. 376, footnote ‡]:

> In U. S. patent 787,412 (April 18, 1905), . . . this remarkable genius clearly outlines the idea of the earth as a resonating circuit (he did not know of the ionosphere), estimates the lowest resonant frequency as 6 Hz (close to the 6.6 Hz for a perfectly conducting sphere), and describes generation and detection of these low-frequency waves. . ..

[33] In an additional short communication on Tesla's discovery of terrestrial stationary waves he writes [*Jackson*, 2003]:

> . . .The belief that Tesla was the first to envision the Earth as an electromagnetic resonator and presage the very low frequency modes of the Earth-ionosphere cavity (now known as Schumann resonances) has been furthered by myself and others. Recently I have learned better.
>
> Tesla's relevant patent application is dated 1905. In a remarkable paper presented in 1893 George F. FitzGerald first observes that the idea of the Earth as a conducting body surrounded by nonconductor is not correct.
>
> He then suggests that the upper regions of our atmosphere are probably fairly good conductors. He models the situation as a conducting sphere surrounded by a concentric conducting spherical shell. . .. Presciently, FitzGerald even mentions thunderstorms, the primary source of Schumann resonance signals, as a source of excitation. . ..

3.5. Hector M. Macdonald's Investigations

[34] In 1902, Hector M. Macdonald (1865–1935), a mathematician and Fellow of Clare College, Cambridge, United Kingdom, worked on the same topic and wrote in chapter 6, "Propagation of electrical effects in simply connected spaces," of his book on *Electric Waves* the following [*Macdonald*, 1902, p. 49]:

> 36. In illustration of the foregoing the possible free oscillations in the space between two concentric spherical surfaces will be considered. . .

and arrived at the following solution for the radially dependent component of the Hertzian potential R_n (after a separation of variables procedure) [*Macdonald*, 1902, p. 52]:

> $$R_n = r^{\frac{1}{2}}\left\{ A J_{n+\frac{1}{2}}(\kappa r) + B J_{-n-\frac{1}{2}}(\kappa r) \right\}.$$
>
> If $r = r_0$ and $r = r_1$ define the bounding surfaces the boundary conditions give
>
> $$A J_{n+\frac{1}{2}}(\kappa r_0) + B J_{-n-\frac{1}{2}}(\kappa r_0) = 0,$$
> $$A J_{n+\frac{1}{2}}(\kappa r_1) + B J_{-n-\frac{1}{2}}(\kappa r_1) = 0,$$
>
> whence
>
> $$J_{n+\frac{1}{2}}(\kappa r_0) J_{-n-\frac{1}{2}}(\kappa r_1) = J_{n+\frac{1}{2}}(\kappa r_1) J_{-n-\frac{1}{2}}(\kappa r_0),$$
>
> an equation to determine κ and thence the possible free periods. When r_0 and r_1 are both finite, this equation has an infinite number of real roots and no others; . . .

Here r is the radial coordinate, κ is the angular wave number, A, B are arbitrary constants, and $J_{n+\frac{1}{2}}$ and $J_{-n-\frac{1}{2}}$ are linearly independent Bessel functions of fractional order, which relate to the functions of order n, nowadays known as spherical Bessel functions of the first kind

$$j_n(z) = \sqrt{\frac{\pi}{2z}} J_{n+\frac{1}{2}}(z)$$

and spherical Bessel functions of the second kind [*Antosiewicz*, 1972, pp. 437 and 438]

$$y_n(z) = \sqrt{\frac{\pi}{2z}} Y_{n+\frac{1}{2}}(z),$$

with the help of the relation

$$y_n(z) = (-1)^{n+1} j_{-n-1}(z).$$

When Macdonald used these functions, the nomenclature (and even the use of commonly agreed symbolic letters) for these special functions had not yet been standardized. Thus one will find different symbols and nomenclature for the same functions in the older literature; e.g., compare with Schelkunoff's investigations (section 3.7).

[35] As Macdonald pointed out, in the case of an ideal cavity resonator there will be an infinite number of resonance frequencies (with decreasing amplitude and spacing as frequency increases). In the real world, the losses in the Earth-ionosphere cavity, the related damping, only allows the first "half-dozen" resonance frequency peaks to be observable above the background noise level. Concerning biographical information about Macdonald the reader is referred to his obituary by Edmund T. Whittaker [*Whittaker*, 1935].

3.6. Investigations by Anton Lampa, Gustav Mie, John W. Nicholson, and Harry Bateman

[36] The first appearance of the problem of oscillations in the cavity between concentric spheres in the German scientific/technical literature on electricity and magnetism can be traced back to an article by the Viennese physics professor Anton Lampa (1868–1938), in the *Proceedings of the Imperial Academy of Sciences in Vienna* (now Austrian Academy of Sciences) in 1903 [*Lampa*, 1903]. In this work he only calculated the oscillation periods of waves inside a sphere, which is embedded in a dielectric, and he compared them with the oscillation periods of a sphere without dielectric cover.

[37] In 1908, Gustav Mie (1868–1957), then professor of physics at the University of Greifswald, in Mecklenburg-Vorpommern, Germany, published his ground-breaking paper about the scattering of light by spherical particles [*Mie*, 1908]. The general solution to this problem also involves cylindrical functions of fractional order (with denominator 2) and is mathematically very similar to the solution of the "spherical cavity oscillations."

[38] Two other early examples of elaborating on the "concentric spheres" problem can be found in the English scientific and technical literature [*Nicholson*, 1906; *Bateman*, 1915]. The mathematician John W. Nicholson (1881–1955), professor of mathematics at King's College, University of London, wrote [*Nicholson*, 1906, p. 707]:

> We proceed to discuss the vibrations between two surfaces of revolution of spherical or spheroidal shape. When they are perfectly conducting, the surface conditions are that the resultant magnetic induction and electric force at the surface are tangential and normal respectively. There are thus two distinct classes of vibrations, corresponding to $\phi = 0$, and $\frac{\partial \phi}{\partial \alpha} = 0$. . ..

[39] Here, ϕ is the potential of the electric field. Nicholson proceeded further:

> Vibrations of Concentric Spheres.
>
> The periods of this simple system have been obtained by Macdonald, as the roots of a complicated transcendental equation. If

> ($r\ \theta\ \omega$) are spherical polar coordinates referred to the centre of the spheres,. . .[the] solution, . . ., is, if $\mu = \cos\theta$,
>
> $$\phi = \sum_{n=0}^{\infty} r^{\frac{1}{2}} \sin^2\theta \frac{dP_n(\mu)}{d\mu} \left\{ AJ_{n+\frac{1}{2}}(kr) + BJ_{-n-\frac{1}{2}}(kr) \right\} \ldots \tag{14}$$
>
> This leads to Macdonald's equation for the periods. . ..

$P_n(x)$ is the Legendre function, k is the angular wave number, and $J_{n+1/2}(x)$ and $J_{-n-1/2}(x)$ are Bessel functions of fractional order.

[40] In the remaining part of his paper, Nicholson considered the case of the oscillations inside a sphere only and did not elaborate on the concentric spheres problem in any more detail.

[41] Harry Bateman (1882–1946) in his classical book on electrical and optical wave motion only mentioned that the problem had been discussed by several authors in the past [*Bateman*, 1915, p. 49]:

> The vibrations . . . for the space between two concentric spheres [have been discussed] by Sir J. J. Thomson (Recent Researches, p. 373.), Sir Joseph Larmor (Proc. London Math. Soc. Ser. 1, Vol. 26 (1894), p. 119.), Prof. H. M. Macdonald (Electric Waves, Chapters 6–7.) and A. Lampa (Wien. Ber. 112 (1903), p. 37.)

but did not go into any details of the calculations.

[42] Harry Bateman received his university education at Cambridge, United Kingdom, emigrated to the United States in 1913, and was later (1917) appointed professor of mathematics, theoretical physics, and aeronautics at the California Institute of Technology, Pasadena.

3.7. Investigations by Sergei A. Schelkunoff

[43] Another important reference to the "concentric spheres" problem can be found in another classical book of electromagnetism, namely, *Electromagnetic Waves* by Sergei A. Schelkunoff (1897–1992). His book was published in 1943, with six reprints following until 1951. Schelkunoff was a Russian emigrant to the United States after World War I who received his university education first at the State College of Washington, Seattle, and later at Columbia University and worked at Bell Telephone Laboratories for three decades afterward [*Schelkunoff*, 1957].

[44] In chapter 10, "Waves, waveguides, and resonators—2" in the section on "Transverse magnetic spherical waves," he derived the solution for the "transverse magnetic (TM) waves" [*Schelkunoff*, 1943, pp. 399–403]:

> We shall now consider fields for which
>
> $$\Pi(r,\theta,\varphi) = T(\theta,\varphi)\hat{T}(r) \ldots . \tag{10–12}$$
>
> If the region is bounded by two perfectly conducting spheres $r = a$ and $r = b$, . . ., thus we have
>
> $$-\frac{B}{A} = \frac{\hat{I}'_n(\sigma a)}{\hat{K}'_n(\sigma a)} = \frac{\hat{I}'_n(\sigma b)}{\hat{K}'_n(\sigma b)}, \quad \frac{\hat{J}'_n(\beta a)}{\hat{N}'_n(\beta a)} = \frac{\hat{J}'_n(\beta b)}{\hat{N}'_n(\beta b)}. \tag{10–29}$$
>
> The first equation applies to dissipative media and the second to non-dissipative media. . .

The prime denotes differentiation with respect to the argument, A, B are arbitrary constants, σ is the angular wave number (can be a complex value in general), and β is its imaginary part. Schelkunoff used the so-called "modified Bessel functions" [*Schelkunoff*, 1943, section 3.5, pp. 51–52]:

$$\begin{aligned} \hat{K}_n(z) &= \left(\frac{\pi z}{2}\right)^{1/2} K_{n+1/2}(z), \\ \hat{J}_n(z) &= \left(\frac{\pi z}{2}\right)^{1/2} J_{n+1/2}(z), \\ \hat{I}_n(z) &= \left(\frac{\pi z}{2}\right)^{1/2} I_{n+1/2}(z), \\ \hat{N}_n(z) &= \left(\frac{\pi z}{2}\right)^{1/2} N_{n+1/2}(z). \end{aligned} \tag{5 – 1}$$

where I_n and K_n relate to the Bessel functions as

$$\begin{aligned} I_n(iz) &= \exp^{in\pi/2} J_n(z), \\ K_n(iz) &= \frac{\pi}{2} \exp^{-i(n+1)\pi/2} [J_n - iN_n] \\ \text{with } N_n(z) &= \frac{J_n(z)\cos n\pi - J_{-n}(z)}{\sin n\pi} \end{aligned}$$

and can be conveniently transformed into J_n and N_n for pure imaginary arguments.

[45] In the next section on "Transverse electric spherical waves" he derived the solution for the "transverse electric (TE) waves" [*Schelkunoff*, 1943, p. 405]:

> Finally, if perfectly conducting spheres $r = a$ and $r = b$ are added,
>
> $$-\frac{B}{A} = \frac{\hat{I}_n(\sigma a)}{\hat{K}_n(\sigma a)} = \frac{\hat{I}_n(\sigma b)}{\hat{K}_n(\sigma b)}, \quad \frac{\hat{J}_n(\beta a)}{\hat{N}_n(\beta a)} = \frac{\hat{J}_n(\beta b)}{\hat{N}_n(\beta b)}. \tag{11–9}$$

[46] In Schelkunoff's book one more section is devoted to the problem of "wave propagation between concentric spheres," where he writes [*Schelkunoff*, 1943, pp. 435–437)]:

> A study of wave propagation between two concentric spheres of nearly equal radii . . . should give us an indication of the magnitude of the curvature effect on the propagation of cylindrical waves between parallel planes. We have seen that in the latter case the principal wave is a uniform cylindrical wave for which the electric lines are straight lines normal to the planes and the magnetic lines circle coaxial with the axis of the wave. The field of the corresponding wave should be independent of φ; and the electric lines should be approximately radial. . ..

> The appropriate solutions can be obtained from the general expressions in section 10. Thus if Π is independent of φ, we have
>
> $$\Pi(r,\theta) = T(\theta)\hat{T}(r), \qquad (23\text{–}3)....$$
>
> The general solution of (3) in terms of the functions best suited to waves in nondissipative media bounded by conducting spheres is
>
> $$\hat{T}(r) = C\hat{J}_n(\beta r) + D\hat{N}_n(\beta r),$$
>
> where $\hat{J}_n$ and $\hat{N}_n$ are defined in section 3.5. [see insert after equation (10–29)]. . . Since E_θ should vanish on the spherical surfaces $r = a$ and $r = b$, we have the following equation
>
> $$-\frac{D}{C} = \frac{\hat{J}'_n(\beta a)}{\hat{N}'_n(\beta a)} = \frac{\hat{J}'_n(\beta b)}{\hat{N}'_n(\beta b)}$$
>
> from which n can be determined. The solution depends on properties of $\hat{J}_n$ and $\hat{N}_n$ regarded as functions of n.

Again C and D are arbitrary constants.

[47] Schelkunoff did not explicitly mention the Earth-ionosphere waveguide as a possible application of his calculations, and he did not calculate the roots of the equations for the resonance frequencies, but he came rather close to the core of the problem.

3.8. Investigations of Olof E. H. Rydbeck

[48] In 1944, Olof E. H. Rydbeck (1911–1999) from Chalmers University of Technology, Gothenburg, Sweden, investigated radio wave propagation problems and published his investigation in the transactions of his university [*Rydbeck*, 1944]. He considered the propagation of radio waves around a spherical Earth surrounded by a concentric layer with a parabolic height distribution of the electron density (the so-called "Epstein layer," named after the physicist Paul Sophus Epstein (1883–1966)) and developed a rather complex set of formulae for a spherical stratified Earth-ionosphere cavity (with four distinct shells).

[49] In a subsequent publication, Rydbeck applied the mathematical formulation to different test cases, among them to a cavity resonator with metallic shells [*Rydbeck*, 1948]. In the course of his calculations Rydbeck also derived the equation to be solved for calculating the resonance frequencies (adapted from Rydbeck's formulae of a very general four-layer model):

$$\frac{\zeta_n^{(1)}(ka)}{\zeta_n^{(2)}(ka)} = \frac{\zeta_n^{(1)}(kb)}{\zeta_n^{(2)}(kb)} \text{ for TE waves}$$

$$\frac{\zeta_n^{(1)'}(ka)}{\zeta_n^{(2)'}(ka)} = \frac{\zeta_n^{(1)'}(kb)}{\zeta_n^{(2)'}(kb)} \text{ for TM waves}$$

$$\text{with } \zeta_n^{(1)}(x) = \sqrt{\frac{\pi x}{2}}H_{n+1/2}^{(1)}(x),$$

$$\text{and } \zeta_n^{(2)}(x) = \sqrt{\frac{\pi x}{2}}H_{n+1/2}^{(2)}(x).$$

The prime denotes differentiation with respect to the argument, k is the angular wave number, and the functions

$$H_n^{(1)}(x) = J_n(x) + iY_n(x)$$
$$H_n^{(2)}(x) = J_n(x) - iY_n(x)$$

are the Hankel functions defined by *Olver* [1972].

[50] Rydbeck did not explicitly calculate the roots of the equations, concentrating on the study of the quality factors of the different models. For an extensive discussion of the quality factor of the Earth-ionosphere waveguide and its consequences for the Schumann resonances, the reader is referred to an early paper by *Jones* [1964].

3.9. Late French Contribution by Jean Broc

[51] The only French reference of the pre-Schumann era (prior to 1952) can be found in a paper by Jean Broc, a doctoral student of Louis de Broglie (1892–1987), published in 1950 [*Broc*, 1950, pp. 198–199] (translation by the present author):

> 1. It is possible to calculate the eigen-frequencies, the field configurations and the quality factor of the resonance modes likely to exist in the spherical space between two concentric spheres (characterized by $s = R_1/R_2$, $R_1 < R_2$).
>
> For the magnetic modes $\mathbf{H}_{mn}$, the eigen-frequencies are found by calculating the roots of the equation
>
> $$(1) \qquad \frac{j_n(x)}{n_n(x)} = \frac{j_n(sx)}{n_n(sx)},$$
>
> where
>
> $$(2) \qquad x = \frac{2\pi R_2}{\lambda},$$
>
> and n is positve and not zero; the functions $j_n(x)$ and $n(x)$ have the known meaning
>
> $$(3) \qquad \begin{cases} j_n(x) = \sqrt{\frac{\pi}{2x}}J_{n+\frac{1}{2}}(x), \\ n_n(x) = \sqrt{\frac{\pi}{2x}}N_{n+\frac{1}{2}}(x). \end{cases} \quad \ldots$$
>
> 2. In the case for the electric modes, the eigen-frequencies are found using the conditions that x must satisfy the condition
>
> $$(5) \qquad \frac{d}{dx}\{(x)[aj_n(x) + bn_n(x)]\} = 0,$$
>
> if one replaces $x = kR_2$ by $sx = kR_1$.

R_1, R_2 are the radii of the inner and outer sphere and j_n and n_n are the spherical Bessel functions of the first and second kind, respectively.

[52] Jean Broc subsequently evaluated the equations for ten different values of the parameter *s* and tabulated the results.

4. Earth-Ionosphere Cavity as a Waveguide

[53] William Thomson (1824–1907), also known later as Lord Kelvin, wrote in an article for *Nichol's Cyclopædia* entitled "Atmospheric electricity" in 1860 (reprinted by *Thomson* [1872, p. 196]):

> ...According to what have been stated above, ... there might be, at any distance considerably exceeding the height of the highest mountain, a uniformly electrified stratum of equal quantity and opposite kind to the earth's, balancing through all the exterior space the force due to the terrestrial electricity, and limiting the manifestations of electric force to the atmosphere within it; ...

This gave the very-well known picture of a giant "spherical capacitor" of the Earth's surface and the conducting strata in the upper atmosphere, which played an important role in interpreting atmospheric electricity phenomena.

[54] Another "mastermind" of conducting atmospheric layers, even predating Thomson, was Carl Friedrich Gauß (1777–1855) in 1839. The concept of a conducting layer surrounding the Earth had been put forward repeatedly by several authors, e.g., in 1883 by Balfour Stewart (1828–1887), in 1889 by Arthur Schuster (1851–1934), and in 1902 by Arthur E. Kennelly (1861–1939) and Oliver Heaviside (1850–1925) independently of each other. For a long time the hypothetical upper atmosphere conducting layer was known as the "Kennelly-Heaviside layer." (The term "ionosphere" was introduced in 1929 and very soon became commonly used.)

[55] In 1921 Charles T. R. Wilson (1869–1959), then reader in electrical meteorology at the University of Cambridge, United Kingdom, developed his concept of a global electrical circuit, consisting of the Earth's surface, the ionosphere, the atmosphere, and the globally (but nonuniformly) distributed thunderstorms, which transmit positive charge by conduction to the upper atmosphere conducting layer and negative charge by cloud-to-ground lightning strokes to the Earth's surface. This DC global electric circuit can also be mathematically described as the zeroth mode of the Schumann resonances, where the electric field is constant and directed radially [*Budden*, 1951; *Schumann*, 1957; *Wait*, 1962].

[56] In 1925 the Briton Edward V. Appleton (1892–1965), then professor of physics at King's College, University of London, and his research student Miles A. F. Barnett (1901–1979), from New Zealand, who was working toward a doctorate at Clare College, Cambridge, United Kingdom, and later became a meteorologist in Australia, published their results obtained at the end of 1924 in an experiment measuring the path length of an indirect ray, proving the existence of the Kennelly-Heaviside layer [*Appleton and Barnett*, 1925]. Their experimental proof was doubted until 1926 when Gregory Breit (1899–1981) together with his research student Merle A. Tuve (1901–1982) published results of their radio echo measurements from the layer using a pulse technique [*Breit and Tuve*, 1926]. Gregory Breit was a Russian-born emigrant to the United States (in 1915), educated in electrical engineering at Johns Hopkins University, Baltimore, and working since 1924 at the Department of Terrestrial Magnetism, Carnegie Institution, Washington, D.C., while the American Merle A. Tuve joined Johns Hopkins University in 1924 to earn a doctorate and collaborated with Breit on the problem of detecting the "Kennelly-Heaviside layer."

[57] Details on the early history of ionospheric research can be found in the comprehensive review by *Green* [1946], the historical development of electromagnetic theory associated with the ionosphere can be found in a review by *Booker* [1974], and reviews of radio propagation theory up to 1950 can be found in publications by *Bremmer* [1948], *Burrows* [1948], and *Booker* [1975].

4.1. George N. Watson's Calculations

[58] In 1918–1919, George N. Watson (1886–1965), then assistant professor of pure mathematics at University College, London, published a paper on the transmission of electric waves around the Earth because his previous calculations of diffraction of electric waves around the Earth did not agree with experimental results. In the paper he stated that [*Watson*, 1918, p. 547]

> ...the absolute value of the Hertzian function [...] is roughly proportional to
>
> $$\sin^{-1/2}\theta \exp\left(-23.94\lambda^{-1/3}\theta\right),$$
>
> where λ is the wavelength measured in kilometers, and θ is the angular distance from the transmitter...
> This formula does not agree with the results obtained experimentally. The numerical factor 23.94 is much too large, ...and it has also been suggested ... [that] the factor $\lambda^{-1/3}$ is replaced by the factor $\lambda^{-1/2}$...
> It... seems desirable to investigate the consequences of the assumption that the Earth is surrounded by a concentric conducting layer at a considerable height.... And it will, in fact, be shown that, if the Earth be regarded as a perfect conductor surrounded by a perfect reflector, the expression for the Hertzian function consists of a finite number of oscillatory terms combined with an infinite series of negative exponentials.

Further on, he calculated the Hertzian function Π near the ground (the radius of the Earth is denoted by *a*, the distance of the transmitter from the center of the Earth is *b*, and the region of the atmosphere which tends to reflect the waves is represented by taking the Earth to be surrounded by a concentric reflector of radius *c*) to be as follows [*Watson*, 1918, p. 550]:

> In the case of a perfect conductor, the hertzian function is
>
> $$\Pi_b^{(0)}(a,\theta) = -\frac{1}{kab}\sum_{n=0}^{\infty}(2n+1)P_n(\mu)\cdot$$
> $$\cdot\frac{\psi_n(kb)\zeta_n'(kc) - \psi_n'(kc)\zeta_n(kb)}{\psi_n'(ka)\zeta_n'(kc) - \psi_n'(kc)\zeta_n'(ka)};$$
>
> and it should be noted that resonance occurs when any of the denominators vanish. As it will be found that the resonance effects do not occur in the actual physical problem (though they may occur in the ideal problem now under consideration), it is convenient to disregard them.

Here k is the angular wave number, $P_n(x)$ is the Legendre function, and $\mu = \cos\theta$, $\psi_n(x)$ and $\zeta_n(x)$ are the spherical Bessel functions of first and second kind, respectively.

[59] In the remainder of his paper Watson did not explicitly calculate the resonance frequencies of the cavity, but his technique to transform the infinite sum of zonal harmonics into a quickly convergent series of residues became known under the name "Watson transformation" and was in use extensively before digital computers were used to solve propagation problems.

4.2. Fritz Noether's Calculations

[60] A publication which closely links to the theory by Watson was worked out by the German physicist and mathematician Fritz Noether (1884–1941). Noether showed that the surface waves (Zenneck waves) cannot be generated by an antenna in a homogeneous atmosphere. Next he considered the possibility of propagation of surface waves between Earth and the Heaviside layer by using a flat (neglecting the Earth's curvature) model and followed Watson's mathematical method mentioned above. As a result of his calculations he came to the conclusion that [*Noether*, 1933, p. 166] (translation by the present author)

> Real guided waves at the Earth could occur only if the conductivity of the Heaviside layer would be larger than or at least equal to the conductivity of the Earth, which is certainly not the case in reality

and he therefore discarded the guided wave theory. Knowledge about the physical conditions of the upper atmosphere was rather limited at that time (1933). In fact, the electrical conductivity of the atmosphere for the ELF frequency range is exponentially increasing from a value of about 10^{-14} Sm^{-1} near the ground to about 10^{-3} Sm^{-1} at an altitude of 100 km, which is roughly the conductivity of the Earth's surface, and increases to values of about 10 Sm^{-1} at an altitude of 200 km [*Bering et al.*, 1998]. This example reveals how a wrong assumption about the value of the conductivity of the ionosphere led Fritz Noether to a wrong conclusion.

4.3. Concept of Waveguide Modes in the Earth-Ionosphere Cavity

[61] The concept of the Earth-ionosphere cavity serving as a waveguide was also used by Anton L. Hales (born 1911), a senior researcher at the Bernhard Price Institute of Geophysical Research, University of Witwatersrand, Johannesburg, South Africa, to explain the slow components of atmospherics (VLF radio waves generated by lightning discharges) measured at the two observation stations in Johannesburg and Durban in 1948 [*Hales*, 1948]. He developed a simple waveguide model, neglecting the Earth's curvature, but he could still qualitatively explain the tail component as a "surface wave." This interpretation could not be maintained further when Wait put forward waveguide dispersion; the lower frequencies arrive later then the VLF atmospherics, as the main cause of the "slow tail" phenomenon [*Wait*, 1960].

[62] Also in 1948 Hendricus Bremmer (1904–1996), a research scientist at the Philips Research Laboratory, Eindhoven, Netherlands, picked up the waveguide concept in connection with very low frequency waves in writing an article on wave propagation [*Bremmer*, 1948, p. 42]:

> For the propagation of very long waves, we may assume the reflections against the earth and the ionosphere as being approximately total, while the ionosphere itself can be considered as sharply defined as it lower boundary at a height h_0 above the earth. . . .We thus arrive at a well-known conception of the ionosphere as a waveguide. . ..

[63] Kenneth G. Budden (1915–2005), then a demonstrator at the Cavendish Laboratory, Cambridge, United Kingdom, expressed his ideas about the Earth-ionosphere cavity waveguide in a publication in 1951 in more detail [*Budden*, 1951, p. 5]:

> §4. Earth and Ionosphere considered as a Waveguide.
>
> Let us first consider the over simplified case where the earth and the ionosphere are both perfect conductors. . .. The elementary theory of waveguides shows that waves can be propagated in a series of different modes. When both surfaces of the guide are perfect conductors, there is one mode in which the electric field is constant across the guide and perpendicular to its surfaces. This mode suffers no attenuation at any frequency. It will be called the mode order zero. Higher-order modes are propagated without attenuation if the frequency is greater than the "cut-off" frequency, which is different for each mode, but for frequencies below the "cut-off" there is heavy attenuation. The "cut-off" frequency depends on the width of the guide, and the order of the mode. . ..We

> We can therefore reject the model using a waveguide with perfectly reflecting surfaces for two reasons: (a) Observations show that at very low frequencies the signal amplitudes from distant atmospherics are much smaller than they would be if an unattenuated zero order mode were present. (b) The observed variation of signal amplitude with frequency is of wrong form. . ..

[64] In a publication of 1952, Budden described the characteristics of the propagating modes in the Earth-ionosphere cavity waveguide [*Budden*, 1952, p. 1184]:

> 2.4. The Waveguide Modes
> The space between the earth and the ionosphere may be regarded as a waveguide of infinite horizontal width, but of finite vertical depth *h*. If the ionosphere is isotropic, then in such a guide modes of two types may be propagated, usually described as TM and TE modes. In the TM modes the magnetic field is everywhere horizontal, and the electric vector is entirely in the plane of propagation. In the TE modes, the electric field is horizontal, and the magnetic field is entirely in the plane of propagation. . ..

5. Schumann's Publications on Electromagnetic Oscillations in the Earth-Ionosphere Cavity

[65] Between 1952 and 1957, Winfried Otto Schumann published about 20 scientific papers on the physics of electrical oscillations in the Earth-ionosphere cavity.

5.1. Publications of 1952

[66] In 1952 Schumann published five articles on the electromagnetic oscillations in the Earth-ionosphere cavity; the first had been submitted on 1 October 1951, and the second had been submitted on 17 January 1952 for publication in *Zeitschrift für Naturforschung* (*Journal for Natural Science Research*) [*Schumann*, 1952a, 1952b]. The third article was submitted on 8 August 1952 to *Zeitschrift für Angewandte Physik* (*Journal for Applied Physics*) [*Schumann*, 1952c]; the fourth, being only a short note, was sent in on 9 August 1952 to *Naturwissenschaften* (*Natural Sciences*) [*Schumann*, 1952d]. The fifth and last article was sent in on 5 October 1952 and was published in December of the same year in the Italian journal *Il Nuovo Cimento* [*Schumann*, 1952e].

[67] The first paper starts with the statement [*Schumann*, 1952a, p. 150] (rough translation by the present author)

> Questions of meteorology, wave propagation and biology are linked with the eigen-oscillations of the Earth with its air film and the enclosing Heaviside-layer. . .. In the following we discuss the situation where a conducting sphere (conductivity = ∞) is surrounded by an air film and the whole ensemble is embedded in an infinitely expanded plasma.

[68] He considered a transverse magnetic (TM) wave, where only the components B_ϕ, E_ρ, E_θ are nonzero, without any dependence on φ, which can be derived from the modulus of the radial vector potential

$$|A| = \sqrt{\rho} H_{n+1/2}(k\rho) P_n(\cos\theta), \qquad k = \omega/c,$$

where ρ is the radial coordinate and $H_{n+1/2}$ is a Bessel function which relates to the spherical Bessel function of the third kind [*Antosiewicz*, 1972]:

$$h_n(z) = \sqrt{\frac{\pi}{2z}} H_{n+1/2},$$

and $P_n(\cos\theta)$ is the Legendre function.

[69] Concerning the lowest eigenfrequency and the source of excitation, he wrote [*Schumann*, 1952a, p. 152]:

> The eigen-frequency can be calculated. . . [and if] we calculate it for the Earth, taking $R \approx 6000$ km and $c = 300000$ km/sec, we calculate $\omega_{\mathrm{ei}} \sim 50\sqrt{2} \sim 70\ \mathrm{sec}^{-1}$ and get the frequency $f_{\mathrm{ei}} \approx 11\ \mathrm{sec}^{-1}$. Through an impulsive excitation of this "cavity", e.g. by a lightning discharge, frequencies of this scale are expected for the lowest excited oscillations.

[70] Next he derived an equation for the wave harmonics:

$$\sqrt{n(n+1)} = \omega_{\mathrm{ei}} R/c.$$

The terms used are *R* for radius of the Earth and *c* for speed of light.

[71] In Schumann's second paper, he deduced an approximation for the attenuation of the lowest eigenfrequencies of the Earth-ionosphere cavity by solving the field equations initially for infinitely conducting boundaries using the Watson transformation. Subsequently, he determined the temporal attenuation factor as the quotient of the mean absorbed heat and averaged total stored electromagnetic energy [*Schumann*, 1952b].

[72] In Schumann's third paper of 1952, he derived approximating formulae for the wave propagation in the Earth-ionsophere cavity, excited by a dipole and with a frequency spectrum of a lightning discharge (Dirac impulse of current), depending on the separation distance (distance from the source), starting from Watson's equations [*Schumann*, 1952c]. The fourth paper is a very short abridged version of the third with only the main equations and conclusions [*Schumann*, 1952d].

[73] The fifth paper of 1952 is an extension of the papers mentioned above, where Schumann calculated the received spectra for different temporal lightning stroke models.

5.2. Schumann's Main Publications Between 1953 and 1957

[74] In the beginning of 1954 Schumann published a paper in which he showed that when calculating the mode

characteristics of long electromagnetic waves in the 1000 km range, the curvature of the Earth could be neglected, and he calculated the propagation properties for two different sources, a horizontal and a vertical electrical dipole [*Schumann*, 1954a]. An extended abstract of this paper was already published in the journal *Naturwissenschaften* at the end of 1953 [*Schumann*, 1953].

[75] Schumann's main publication of 1954 was a two-part paper in which he elaborated on the radial Hertz potentials of a horizontal source dipole in the spherical cavity and the corresponding transverse magnetic and electric waves [*Schumann*, 1954b, 1954c]. He also mentioned for the first time that the attenuation increases with frequency, and therefore he expected only the lowest resonance frequency to be measurable.

[76] Schumann's last paper connected to the Earth-ionosphere cavity oscillations dates from 1957, in which he reported the calculation of the field intensities at the resonance frequencies and the influence of the skin effect of the ionosphere on the propagation properties [*Schumann*, 1957]. Schumann also emphasized that losses in the system have two consequences: The first is that the resonance frequencies change to lower values, and the second is that at higher resonance frequencies, the resonance effects are diminished.

[77] In addition to Schumann's publications, the following publications of students and colleagues from the Electro-Physical Laboratory at the Technische Hochschule München are worth mentioning. Julius Weidner, one of Schumann's doctoral students, investigated in 1954 the effect of considering one additional term for the Watson transformation but did not come to a conclusive answer concerning an improvement of the solution [*Weidner*, 1955]. As a supplement to the publication [*Schumann*, 1954a], his student Hans-Georg Stäblein published a paper in which he calculated the time-dependent behavior of signals from lightning strokes with currents of exponential form [*Stäblein*, 1955]. In 1957, Ortwin Rösner, a scientific assistant at the Physical Institute Weienstephan (about 70 km northeast of Munich, near Landshut, Bavaria), worked out a study under Schumann's guidance concerning the propagation and resonance behavior of TE waves in the Earth-ionosphere cavity [*Rösner*, 1957].

5.3. Experimental Work by Herbert L. König

[78] In 1954, Winfried Otto Schumann, together with his doctoral student Herbert L. König (1925–1996), published preliminary results of their observations of extremely low frequencies atmospherics at their institute in Munich and at a reference station outside the populated area and far from the disturbing electrical railway. König implemented the experimental setup, and measurements with a day-long integrating recorder were performed. They could distinguish two main groups of registered waveforms, one being of almost sinusoidal character with a frequency of about 9 Hz [*Schumann and König*, 1954].

[79] In 1959, König published an abridged version of his doctoral thesis in the German journal *Zeitschrift für Angewandte Physik*, where he could confirm the earlier published results of *Schumann and König* [1954]. The only modification was that the measured frequency of 9 Hz of the almost sinusoidal-type oscillations shifted to lower frequencies (8–9 Hz). In addition, he could also show that the intensity of the lowest mode exhibits diurnal variations, but he could not derive any spectral representations for the frequency range under study because of the lack of a fancy spectral analyzer [*König*, 1959].

[80] In March 1960, König communicated his findings to the international scientific community at the 1960 Institute of Radio Engineers International Convention taking place in the Waldorf-Astoria Hotel, New York, in a talk on "Ultra-low-frequency atmospherics" [*König*, 1960]. Already in this publication, König elaborated on a topic which he was very interested in during the years to come, namely, the effects of ELF atmospherics on biological systems and humans (the field is nowadays often called "bioelectromagnetics"). His interest in and work on this subject culminated in 1975 with the publication of his book *The Invisible Environment: The Human Being Influenced by Electromagnetic Interactions*, which he published in his own publishing company in five printings until 1986 [*König*, 1975].

[81] Starting in 1960 and extending for almost a decade, König and Charles Polk (1919–1999), from the University of Rhode Island, Kingston, performed continuous measurements of Schumann resonances at their respective institutes (abstracted by *König* [1975]). König continued his experimental work at the institute after the theoretician Schumann retired in 1961. Most of König's measurements were taken at his summer residence in Brannenburg, Inn Valey, southern Bavaria (about 20 km south of Rosenheim), which was chosen because the electromagnetic noise level of the inner city of Munich disabled useful measurements at the Electrophysical Institute (described in the recollections of König's son [*König*, 2005]). In 1966 König qualified for university teaching at the Technische Universität München, and in 1974 he was promoted to adjunct professor. He retired in 1990. For a short biographical sketch on König, the reader is referred to a short eulogy by Charles Polk given in 2000 at the occasion of a conference in Munich [*Polk*, 2000].

5.4. Transverse Resonances of the Earth-Ionosphere Waveguide

[82] In addition to the Schumann resonances, which would be depicted as azimuthal resonances, there are

also transverse resonances of the Earth-ionosphere cavity. These are related to the radial extent of the waveguide. The first ideas could be read between the lines of papers by *Budden* [1951] (see section 4.3). The first specific calculations of the transverse resonances were performed in 1961 by Hermann Poeverlein (1911–2002), one of Schumann's former colleagues at the Electrophysical Institute (from 1942 to 1953) [*Poeverlein*, 1961]. Poeverlein worked between 1953 and 1969 at the Cambridge Research Laboratory, Bedford, Massachusetts, of the United States Air Force, before being appointed professor at the Technische Hochschule Darmstadt (since 1997 named Technische Universität Darmstadt (Darmstadt University of Technology), Germany.

6. Reception of Schumann's Publications by the Scientific Community in the First Decade

6.1. Introductory Remarks

[83] In this section we discuss the reception of Schumann's early publications on the Earth-ionosphere cavity waveguide oscillations in the ELF range by his contemporaries and the scientific community at large. The period under investigation of about 10 years was chosen rather arbitrarily but at last can be justified by the fact that the term "Schumann resonances" was coined in 1962 (see below) and then became more and more popular in the scientific literature thereafter.

[84] Schumann's publications were all written in German, and we could not find any citations, other than in his own publications or those of his students, up to 1956. Because of the lack of a universal citation index for German scientific literature, we could only scan some appropriate journals. For the reception in the English scientific-technical literature we gathered information by scanning the two 10 year cumulative indices of the *Science Citation Index* for the periods 1945–1954 (published in 1988) and 1955–1964 (published in 1984). Knowing of the publication practice of that time, where there existed only a limited number of journals in the field and many new findings and discoveries were first published in conference proceedings or even textbooks, a more comprehensive search has been carried out in the appropriate journal literature (since some of the journals were not included in the *Citation Index*) and books on related subjects.

6.2. First Citations and Appreciations

[85] The first citation of any of Schumann's Earth-ionosphere cavity waveguide publications could be identified in an article by Jerry Shmoys (born 1923) of the Department of Electrical Engineering, Polytechnic Institute of Brooklyn, Brooklyn, New York (since 1973 part of Polytechnic Institute of New York, later renamed Polytechnic University) entitled "Long-range propagation of low-frequency radio waves between the Earth and the ionosphere" and published in the February issue of the *Proceedings of the Institute of Radio Engineers* [*Shmoys*, 1956]. Shmoys considered electromagnetic mode propagation between a perfectly conducting (flat) Earth and a gradually varying ionosphere. He mentioned in the introduction earlier publications taking into account a sharp discontinuity between the atmosphere and ionosphere, among them Schumann's papers of 1954 [*Schumann*, 1954a, 1954b, 1954c].

[86] The first citation of Schumann's papers of 1952 can be found in a publication by Leonard Liebermann from the Marine Physical Laboratory of the Scripps Institution of Oceanography at the University of California, La Jolla, California, on propagation properties of ELF electromagnetic waves [*Liebermann*, 1956b]. He derived a Fourier integral relationship between the received spectrum and the shape of the radiated spectrum from lightning and mentioned that Schumann obtained a similar expression by another approach not based on waveguide theory [*Schumann*, 1952d].

[87] The first citation of one of Schumann's *Zeitschrift für Naturforschung* papers of 1952 [*Schumann*, 1952b] can be found in a publication by Martin Balser and Charles A. Wagner from the Lincoln Laboratory of the Massachusetts Institute of Technology [*Balser and Wagner*, 1960a], showing spectra of radio noise from 50 to 100 Hz, published in July/August 1960, a few months prior to their publication in *Nature* [*Balser and Wagner*, 1960b].

6.3. Reception in the USSR

[88] As a prefix to this subsection it must be admitted that the search in Russian-language scientific publications was very limited because of their inaccessibility and therefore does not claim to be comprehensive at all. For a summary of radio wave propagation research in the USSR up to the dawn of the space age (about 1955) we refer to the review by *Kazantsev* [1957].

[89] An early reference to a publication by Schumann can be found in a review on ultralong radio wave propagation (10 Hz to 50 kHz) by *Borodina et al.* [1960] from the Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Academy of Sciences of the USSR, Moscow. The authors cite both of Schumann's publications of 1954 in connection with his calculations of the field strength at the antipode of a lightning source [*Schumann*, 1954b, 1954c, 1954d] as well as the ELF measurements performed by Schumann and his student König in 1954 [*Schumann and König*, 1954], emphasizing the importance of their findings for

the research of ultralow frequency wave propagation in general [*Borodina et al.*, 1960, p. 27].

[90] At about the same time, the later to be famous Russian physicist Vitaly L. Ginzburg (born 1916) published in his book on *Propagation of Electromagnetic Waves in Plasma*, which was soon afterward translated into English [*Ginzburg*, 1961], a passing mention of Schumann's paper of 1955 [*Schumann*, 1955].

[91] A brief mention of Schumann's papers of 1952 and 1954 [*Schumann*, 1952d, 1954a, 1954b, 1954c, 1954d] also appeared in the classical book by Yakov L. Alpert (born 1911) on *Radio Wave Propagation and the Ionosphere*, published in 1960 and translated into English in 1963 [*Alpert*, 1963]. However, Schumann's work did not get proper credit within these classical books, and even in his recent autobiography Alpert touched on Schumann's work only in connection with the above-mentioned "antipode effect" when he stated [*Alpert*, 2000, pp. 133–134]:

> . . .we found experimentally from the records of *Sputnik*'s radio signals the Antipode Effect, as it is known in the scientific literature. This phenomenon was theoretically predicted by the German physicist Wolf [sic] Schumann in 1952. . ..

[92] Petr E. Krasnushkin (1913–1983), from the Steklov Mathematical Institute of the Soviet Academy of Sciences, cited Schumann's work of 1954 [*Schumann*, 1954b, 1954c] in a paper on the theory of terrestrial atmospherics [*Krasnushkin*, 1962a]. Finally, Schumann's work got proper credit by Krasnushkin at the end of 1962, when he referred to his publications [*Schumann*, 1952d, 1952e, 1954a, 1954b, 1954c] in a review in English for the *Supplements to Nuovo Cimento* [*Krasnushkin*, 1962b]. Also in 1962 a group of the Institute of Physics of the Earth of the Soviet Academy of Sciences, Moscow, referred to Schumann's measurements [*Vladimirov and Kleimenova*, 1962] by citing the paper of *Schumann and König* [1954].

6.4. Term "Schumann Resonances"

[93] The first publication introducing the term Schumann resonances (even in the title of the paper) to the lowest resonance oscillations of the Earth-ionosphere cavity could be traced back to a paper by Charles Polk and Franklin Fitchen, both working at the Department of Electrical Engineering, University of Rhode Island, Kingston, in 1962 [*Polk and Fitchen*, 1962].

[94] Concerning the first introduction of the term "Schumann resonances" into the scientific/technical literature, see also the following quote from a letter to the author by Earle R. Williams (private communication, April 2006): "About 10 years ago, Charles Polk confirmed to me his first use in the scientific literature of the term ''Schumann resonances'' consistent with the author's educated guess." Charles Polk, an Austrian-born emigrant after Austria's annexation by Hitler's Germany in 1938, finally came to the United States in 1940 and received his doctorate in electrical engineering from the University of Pennsylvania. He joined the University of Rhode Island in 1959 and specialized in the biological effects of electric and magnetic fields [*Bioelectromagnetics Society Newsletter*, 2000].

[95] Charles Polk also suggested to one of his students, Mian M. Abbas, to study the possibility of excitation of Schumann resonances by hydromagnetic waves generated in the magnetosphere and propagated through the ionosphere. He worked out this idea in his doctoral thesis (1966) under the guidance of Charles Polk and Hermann Poeverlein and published the results with its positive conclusions [*Abbas*, 1968].

[96] The term "Earth-ionosphere cavity resonances" was finally superseded by "Schumann resonances" in 1965 after the publication of the historical review by Theodore R. Madden (born 1925) and W. J. Thompson in May, in which they stated [*Madden and Thompson*, 1965, p. 212]:

> It is not until 1952, however, that we find any literature concerning itself [Earth-ionosphere waveguide] with the resonance of the entire waveguide system. *Schumann* was the first to study the theoretical aspects of the problem;. . . For this reason the phenomena of the Earth-ionosphere cavity resonances are known as the Schumann resonances.

[97] Subsequently, two more influential reviews with the term "Schumann resonances" in their titles were published in 1965 and paved the way for its increasing use in the scientific/technical literature. One appeared in August and was by Janis Galejs, working at the Applied Research Laboratory of Sylvania Electronic Systems, Waltham, Massachusetts [*Galejs*, 1965], and the other appeared in October and was by R. K. Cole, from the Stanford Research Institute at Menlo Park, California [*Cole*, 1965].

[98] Janis Galejs (1927–1972) was a Latvian-born emigrant who graduated from the Technische Hochschule Braunschweig, Germany, in 1950. After coming to the United States at the end of 1950 he enrolled again in university studies and received his doctorate in 1957 from the Illinois Institute of Technology, Chicago. During his professional career he worked for several military research laboratories and finally at the Communication Sciences Division of the Naval Research Laboratory in Washington, D.C. [*Wait*, 1974b]. Until his early death he contributed several outstanding and trend-setting publications to the ELF electromagnetic wave propagation theory, one being his famous and well-known book on

Terrestrial Propagation of Long Electromagnetic Waves [*Galejs*, 1972].

7. ELF Wave Measurements of the Early Days

7.1. ELF Measurements in the Pre-Schumann Era

[99] Observations of ELF waves were not entirely new at the time when Schumann proposed the resonances of the Earth-ionosphere cavity waveguide in 1952. Already in 1937 a group at the British Radio Department of the National Physics Laboratory [*Watson-Watt et al.*, 1937, pp. 271–272] described results of their observations of the waveform of atmospherics in a paper where they stated that "In very few cases were "slow" forms recorded without traces of an accompanying "oscillatory" form…." These "slow tails" (about 100–500 Hz) of atmospherics showed up in observations, but the receivers did not receive well in the ELF band (below about 100 Hz) at that time.

[100] Influenced by the discovery of solar radio noise in the 30–1000 MHz range, Donald H. Menzel (1901–1976) and Winfield W. Salisbury in 1948 raised the question whether lower frequencies may also exist. They postulated the existence of frequencies in the range of 1–500 Hz and that these long waves would be diffracted around the Earth. To validate their suggestions, they required experimental verification [*Menzel and Salisbury*, 1948]. In a subsequent paper, H. F. Willis of the British Admiralty Research Laboratory reported measurements of magnetic fluctuations in the 5–1000 Hz range, but no frequency characteristics could be detected in the 5–100 Hz range, mainly because of the large bandwidth of the receiver; nor could the measurements shed light on the real source of these waves [*Willis*, 1948]. One of the reasons why the measurements had not been as significant as expected was that only analog filtering was used at that time, whereas advanced spectral techniques in hardware became available only at the end of the 1950s and beginning of the 1960s.

[101] In 1951, Jules Aarons (born 1921) reported on noise measurements in the frequency band 0.5–15 Hz taken by the Upper Air Laboratory of the Geophysical Directorate of the U.S. Air Force made more than one year earlier. Concerning the cause of the varying low-frequency noise, he stated [*Aarons*, 1951, p. 278]:

> It is suggested that the variable component of the low-frequency noise, including the bursts and increased noise level, may lie in plasma-like oscillations of the ionosphere, which may generate the low frequencies…. In the case of low-frequency fluctuations, the signal comes directly from the plasma-like oscillations.

Just one year later, Schumann proposed that the global resonances of the Earth-ionosphere waveguide would be responsible for signals in this frequency range, and after the definitive proof of their existence in 1960 Aarons' argument was refuted.

7.2. ELF Wave Measurements in the First Decade After Schumann's Publications

[102] The results of the early measurements of Schumann's student Herbert L. König in 1954 at Schumann's Institute are reported in section 5.3. However, other groups, especially at the University of California at Los Angeles [*Holzer and Deal*, 1956] and at La Jolla [*Liebermann*, 1956a, 1956b] investigated the predicted ELF electromagnetic waves. They summarized their results as follows [*Holzer and Deal*, 1956, p. 537]:

> While the results presented here are not in themselves conclusive evidence, they strongly suggest that the majority of the electromagnetic signals between 25 and 130 cycles/sec. are atmospherics. Further, the signals in this frequency-range appear to have a sufficiently low attenuation to suggest that they are received from lightning in all parts of the world. In the case of properly selected observing sites, remote from large thunderstorm centres, the mean signal amplitude [should be "the mean squared signal amplitude" (Earle R. Williams, private communication, April 2006)] is roughly proportional to the number of storms in progress. Thus it appears possible that an index of world thunderstorm activity may be obtained from one or a very small number of carefully selected low-frequency stations.

[103] Leonard Liebermann of the Scripps Institution of Oceanography in La Jolla stressed the point of variability in his conclusions [*Liebermann*, 1956b, p. 1483]:

> Extremely low frequency propagation has in common with higher transmission frequencies, a familiar characteristic: variability. At times, extremely low frequencies propagate as if in a "modified" waveguide; at other times the propagation mechanism is altered, resulting in the "anomalous" propagation described above. These variability in propagation undoubtedly can be ascribed to variations in the electrical properties of the ionosphere.

In his conclusions Lieberman did not consider the inherent variability of the source (lightning strokes), which, in further studies, turned out to be responsible for most of the differences between received signals. For the results of König's measurements at Schumann's institute and in the surroundings of Munich prior to 1959 the reader is referred to section 5.3.

[104] Near the end of 1960 Martin Balser and Charles A. Wagner, from the Lincoln Laboratory of the Massachusetts Institute of Technology, were the first to publish representative spectra of the frequency range from 5 to 34 Hz showing the first five modes quite clearly [*Balser and Wagner*, 1960b]. About the attempts

leading to the successful measurements, a reminiscence of one of the persons involved is quoted from a letter to the author by Earle R. Williams (private communication, April 2006):

> Charles Wagner summarized the history of their efforts to detect the Schumann resonances in New England in a conversation we had several years ago. Three efforts were mounted. The first attempt took place on a boat in Boston harbor, with an electrode on the upper mast, but there was so much motion of the boat and the mast, that no clear signals could be seen. The second effort took place along the Kancamagus Highway in New Hampshire, and then they got their first glimpse of the signals, but they were still not sufficiently content to publish the results. Finally, they placed an electrode on a rigid metal tower in Ipswich, Massachusetts and got beautiful frequency spectra. They had a big party that night. . . .

[105] In 1962, Roger Gendrin and Robert Stefant, from the Service d'Aéronomie of the French Centre National de la Recherche Scientifique at Verrières-le-Buissson, a suburb of Paris, reported measurements of Schumann resonance spectra and how they were influenced by a high-altitude nuclear explosion [*Gendrin and Stefant*, 1962a, 1962b]. For an extensive list of early experimental Schumann resonance investigations we refer to Table 6.1 in the book by *Nickolaenko and Hayakawa* [2002].

[106] With the confirmation of their experimental results by several other independent groups [e.g., *Chapman and Jones*, 1964; *Rycroft*, 1965], the existence of Earth-ionosphere cavity waveguide oscillations proposed in 1952 by Winfried Otto Schumann was beyond doubt. Subsequently, theoretical and experimental investigations of ELF waves have developed further in the 1960s, and the reader is referred to the literature reviews mentioned in section 1 for details of the historical development.

[107] **Acknowledgments.** The author would like to thank all his friends and colleagues, who allocated their valuable time for discussing parts of the study with him. In particular, the author is in debt to Arcadi V. Usmanov (now at the NASA Goddard Space Flight Center) for his help in locating several hard-to-find documents in the GSFC library; to Siegfried J. Bauer and Reinhart Leitinger, University of Graz, for numerous discussions; to Martin Friedrich, Graz University of Technology; Juan A. Morente, University of Granada; and Hans Eichelberger, Zoltan Vörös, and Herbert Lichtenegger, from the author's home institution, for their careful reading and their numerous valuable comments on earlier versions of the manuscript. Special thanks are due to J. David Jackson, University of California, Berkeley, for his unselfish help in throwing light on the author's questions associated with FitzGerald and Tesla. In addition, the author thanks members of Team 85: Titan's Ground-Ionosphere Cavity after Huygens: Atmospheric Electricity and Surface for fruitful discussions on some aspects of the topic during team meetings in the International Space Science Institute, Bern, Switzerland. The author thanks the three referees for their comments, which helped to improve the manuscript considerably.

References

Aarons, J. (1951), Low-frequency noise disturbance, *Proc. Am. Acad. Arts Sci.*, *79*, 266–279.

Abbas, M. (1968), Hydromagnetic wave propagation and excitation of Schumann resonances, *Planet. Space Sci.*, *16*, 831–844.

Al'pert, Y. L. (1963), *Radio Wave Propagation and the Ionosphere*, 364 pp., Consult. Bur., New York.

Alpert, Y. L. (2000), *Making Waves: Stories From My Life*, 260 pp., Yale Univ. Press, New Haven, Conn.

Antosiewicz, H. A. (1972), Bessel functions of fractional order, in *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, pp. 435–478, Dover, Mineola, N. Y.

Appleton, E. (1961), Sir Joseph Larmor and the ionosphere, *Proc. R. Irish Acad., Ser. A*, *61*, 55–66.

Appleton, E. V., and M. A. F. Barnett (1925), On some direct evidence for downward atmospheric reflection of electric rays, *Proc. R. Soc. London, Ser. A*, *109*, 621–641.

Balser, M., and C. A. Wagner (1960a), Measurements of the spectrum of radio noise from 50 to 100 cycles per second, *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *64*, 415–419.

Balser, M., and C. A. Wagner (1960b), Observations of Earth-ionosphere cavity resonances, *Nature*, *188*, 638–641.

Barr, R., D. Llanwyn Jones, and C. J. Rodger (2000), ELF and VLF radio waves, *J. Atmos. Sol. Terr. Phys.*, *62*, 1689–1718.

Bateman, H. (1915), *The Mathematical Analysis of Electrical and Optical Wave-Motion on the Basis of Maxwell's Equations*, 159 pp., Cambridge Univ. Press, Cambridge, U. K.

Bering, E. A., III, A. A. Few, and J. R. Benbrook (1998), The global electric circuit, *Phys. Today*, *51*(10), 24–30.

Bioelectromagnetics Society Newsletter (2000), In memoriam Charles Polk, *156*, *Bioelectromagnetics Society Newsletter*, 1–3.

Blackband, W. T. (Ed.) (1964), *Propagation of Radio Waves at Frequencies Below 300 kc/s*, 478 pp., Pergamon, Oxford, U. K.

Bliokh, P. V., A. P. Nickolaenko, and Y. F. Fillippov (1980), *Schumann Resonances in the Earth-Ionosphere Cavity*, *IEE Electromagn. Wave Ser.*, vol. 9, 166 pp., Peter Peregrinus, London.

Booker, H. G. (1974), Fifty years of the ionosphere. The early years—Electromagnetic theory, *J. Atmos. Terr. Phys.*, *36*, 2113–2136.

Booker, H. G. (1975), Developments in the theory of radio propagation, 1900–1950, *Radio Sci.*, *10*(7), 665–668.

Borodina, S. V., Y. K. Kalinin, G. A. Mihaylova, and D. S. Fligel (1960), Obzor sovremennogo sostoyaniya issledovaniy rasprstraneniya sverhlinnih elektromagntnih voln (Review of the contemporary status of research on the propagation of ultra-long electromagnetic waves), *Izv. Vyssh. Uchebn. Zaved. Radiofiz.*, *3*(1), 5–32.

Breit, G., and M. A. Tuve (1926), A test of the existence of the conducting layer, *Phys. Rev.*, *28*, 554–573.

Bremmer, H. (1948), General remarks on the theory of wave propagation, in *Atti del Congresso Internazionale per il Cinquantenario della Scoperta Marconiana della Radio*, pp. 30–42, Dott. Giovanni Bardi, Rome.

Broc, J. (1950), Sur les caractéristiques des cavités résonnantes électromagnétiques constituées par deux sphères concentriques (On the characteristiques of the electromagnetic resonant cavity constituted by two concentric spheres), *C. R. Hebd. Seances Acad. Sci.*, *230*, 198–199.

Budden, K. G. (1951), The propagation of a radio-atmospheric, *Philos. Mag.*, *42*, 1–19.

Budden, K. G. (1952), The propagation of a radio atmospheric—II., *Philos. Mag.*, *43*, 1179–1200.

Budden, K. G. (1961), *The Wave-Guide Mode Theory of Wave Propagation*, 325 pp., Logos, London.

Burrows, C. R. (1948), Mechanisms of propagation, in *Atti del Congresso Internazionale per il Cinquantenario della Scoperta Marconiana della Radio*, pp. 43–51, Dott. Giovanni Bardi, Rome.

Burrows, M. L. (1978), *ELF Communications Antennas*, *IEE Electromagn. Wave Ser.*, vol. 5, 245 pp., Peter Peregrinus, London.

Chapman, F. W., and D. L. Jones (1964), Observations of Earth-ionosphere cavity resonances and their interpretation in terms of a two-layer ionosphere model, *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *68*, 1177–1185.

Cole, R. K. (1965), The Schumann resonances, *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *69*, 1345–1349.

Eckart, G. (1975), Winfried Otto Schumann, *Arch. Elektron. Uebertrag.*, *29*, 52.

Fischer, E. P. (2006), Fremde Federn. Kolumne: Im Gegenteil (Adorn with borrowed plumes. Column: On the contrary), *Welt*, 27, 24 July. (Available online at http://www.welt.de/data/2006/07/24/970463.html.)

FitzGerald, G. F. (1893), On the period of vibration of electrical disturbances upon the Earth, in *Rep. Meet. Br. Assoc. Adv. Sci.*, *63*, 682.

FitzGerald, G. F. (1902), *The Scientific Writings of the Late George Francis FitzGerald*, edited by J. Larmor, 576 pp., Hodges, Figgis, Dublin.

Galejs, J. (1965), Schumann resonances, *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *69*, 1043–1055.

Galejs, J. (1972), *Terrestrial Propagation of Long Electromagnetic Waves*, 362 pp., Pergamon, Oxford, U. K.

Gendrin, R., and R. Stefant (1962a), Effet de l'explosion thermonucléaire à très haute altitude du 9 juillet 1962 sur la résonance de la cavité terre-ionosphère. Résultats expérimentaux (Effect of high altitude thermonuclear explosion on 9 July 1962 on the resonance of the Earth-ionosphere cavity. Experimental results), *C. R. Hebd. Seances. Acad. Sci.*, *255*, 2273–2275.

Gendrin, R., and R. Stefant (1962b), Effet de l'explosion thermonucléaire à très haute altitude du 9 juillet 1962 sur la résonance de la cavité terre-ionosphère. Interprétation (Effect of high altitude thermonuclear explosion on 9 July 1962 on the resonance of the Earth-ionosphere cavity. Interpretation), *C. R. Hebd. Seances. Acad. Sci.*, *255*, 2493–2495.

Ginzburg, V. L. (1961), *Propagation of Electromagnetic Waves in Plasma*, 822 pp., Gordon and Breach, New York.

Green, A. L. (1946), Early history of the ionosphere, *Amalgamated Wireless Australasia Tech. Rev.*, *7*(2), 177–228.

Hales, A. L. (1948), A possible mode of propagation of the 'slow' or tail component in atmospherics, *Proc. R. Soc. London, Ser. A*, *193*, 60–71.

Heilbron, J. L. (1976), Thomson, Joseph John, in *Dictionary of Scientific Biography*, vol. 13, edited by C. C. Gillispie, pp. 362–372, Scribner, New York.

Holzer, R. E., and O. E. Deal (1956), Low audio-frequency electromagnetic signals of natural origin, *Nature*, *177*, 536–537.

Jackson, J. D. (1998), *Classical Electrodynamics*, 3rd ed., 808 pp., John Wiley, Hoboken, N. J.

Jackson, J. D. (2003), FitzGerald's observation preceded Tesla's, *Am. Phys. Soc. News*, *17*(7), 4.

Jones, D. L. (1964), The calculation of the *Q* factors and frequencies of Earth-ionosphere cavity resonances for a two-layer model, *J. Geophys. Res.*, *69*(19), 4037–4046.

Kazantsev, A. N. (1957), U.S.S.R research into radio wave propagation in the ionosphere, *Radio Eng. Electron.*, Engl. Transl., *2*(11), 62–83.

Killy, W. (Ed.) (1995), Arnold, Engelbert, in *Deutsche Biographische Enzyklopädie*, vol. 1, p. 186, K. G. Saur, Munich, Germany.

Killy, W., and R. Vierhaus (Eds.) (1998), Schumann, Winfried (Otto), in *Deutsche Biographische Enzyklopädie*, vol. 9, p. 208, K. G. Saur, Munich, Germany.

Knoll, M. (1968), Zum 80. Geburtstag von W. O. Schumann (To the 80th birthday of W. O. Schumann), *Z. Angew. Phys.*, *25*, 1.

König, F. M. (2005), Ein historischer Rückblick "in die Zukunft" um die Schumann-Resonanz (A historical review "into the future" about the Schumann resonance), *Wetter Boden Mensch*, no. 2, 13–24.

König, H. (1959), Atmospherics geringster Frequenzen (Atmospherics of lowest frequencies), *Z. Angew. Phys.*, *11*, 264–274.

König, H. (1960), Ultra-low-frequency atmospherics, in *IRE International Convention Record*, vol. 8, part 1, pp. 128–133, Inst. of Electr. and Electron. Eng., New York.

König, H. L. (1975), *Unsichtbare Umwelt. Der Mensch im Spielfeld elektromagnetischer Kräfte (The Invisible Environment. The Human Being Influenced by Electromagnetic Interactions)*, 203 pp., Herbert L. Koenig, Munich, Germany.

Krasnushkin, P. E. (1962a), K teorii zemnih atmosferikov (On the theory of terrestrial atmospherics), *Dokl. Akad. Nauk SSSR*, *144*(2), 334–337.

Krasnushkin, P. E. (1962b), On the propagation of long and very long waves around the Earth, *Nuovo Cimento Suppl. Ser. 10*, *26*(1), 50–112.

Lampa, A. (1903), Über die elektromagnetischen Schwingungen einer Kugel sowie über diejenigen einer Kugel, welche von einer konzentrischen dielektrischen Kugelschale umgeben ist (On the electromagnetic oscillations of a sphere as well as those of a sphere, which is surrounded by a concentric dielectric spherical shell), *Sitzungsber. Math. Naturwiss. Kl. Kaiserliche Akad. Wiss. Wien*, *112*, part IIa, 37–66.

Larmor, J. (1929), Electric vibrations in condensing dielectric systems, in *Mathematical and Physical Papers*, vol. 1, pp. 356–378, Cambridge Univ. Press, Cambridge, U. K.

Liebermann, L. (1956a), Extremely low-frequency electromagnetic waves. I. Reception from lightning, *J. Appl. Phys.*, *27*(12), 1473–1476.

Liebermann, L. (1956b), Extremely low-frequency electromagnetic waves. II. Propagation properties, *J. Appl. Phys.*, *27*(12), 1477–1483.

Macdonald, H. M. (1902), *Electric Waves*, 200 pp., Cambridge Univ. Press, Cambridge, U. K.

Madden, T., and W. Thompson (1965), Low-frequency electromagnetic oscillations of the Earth-ionosphere cavity, *Rev. Geophys.*, *3*, 211–254.

Medicus, G. (1948), Zum 60. Geburtstag von Prof. Dr.-Ing. W. O. Schumann am 20. Mai 1948 (To the 60th birthday of W. O. Schumann), *Z. Angew. Phys.*, *1*, 105–106.

Menzel, D. H., and W. W. Salisbury (1948), Audio-frequency radio waves from the Sun, *Nature*, *161*, 91.

Mie, G. (1908), Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen (Contributions to the optics of turbid media, particularly of colloidal metal solutions), *Ann. Phys.*, *25*, 377–445.

Nature (1893), The period of vibration of disturbances of electrification of the Earth, *Nature*, *48*, 526–529.

Nicholson, J. W. (1906), The symmetrical vibrations of conducting surfaces of revolution, *Philos. Mag.*, *11*, 703–721.

Nickolaenko, A. P., and M. Hayakawa (2002), *Resonances in the Earth-Ionosphere cavity*, 380 pp., Kluwer Acad., Doordrecht, Netherlands.

Noether, F. (1933), Bemerkungen über das Ausbreitungsgesetz für lange Wellen und die Wirkung der Heavisideschicht (Comments about the propagation law for long electrical waves and the effect of the Heaviside-layer), *Elektr. Nachrichtentech.*, *10*, 160–172.

Olver, F. W. J. (1972), Bessel functions of integer order, in *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, pp. 355–433, Dover, Mineola, N. Y.

Piloty, H. (1958), W. O. Schumann zum 70. Geburtstag (W. O. Schumann dedicated to his 70th birthday), *Elektrotechn. Z.*, *79*, 369–371.

Poeverlein, H. (1961), Resonance of the space between Earth and ionosphere, *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *65*, 465–473.

Poggendorff, J. C. (Ed.) (1904), Schumann, Ernst Otto, in *J. C. Poggendorffs Biographisch-Literarisches Handwörterbuch zur Geschichte der Exacten Wissenschaften*, vol. 4, part 2, pp. 1365–1366, Verl. Barth, Leipzig, Germany.

Poggendorff, J. C. (Ed.) (1940), Schumann, Winfried Otto, in *J. C. Poggendorffs Biographisch-Literarisches Handwörterbuch*, vol. 6, part 4, p. 2386, Verl. Chem., Berlin.

Poggendorff, J. C. (Ed.) (1956), Arnold, Engelbert, in *J. C. Poggendorffs Biographisch-Literarisches Handwörterbuch zur Geschichte der Exacten Naturwissenschaften*, vol. 7a, part 1, p. 57, Akademie, Berlin.

Poggendorff, J. C. (Ed.) (1961), Schumann, Winfried Otto, in *J. C. Poggendorffs Biographisch-Literarisches Handwörterbuch zur Geschichte der Exacten Naturwissenschaften*, vol. 7a, part 4, pp. 329–330, Akademie, Berlin.

Polk, C. (1982), Schumann resonances, in *CRC Handbook of Atmospherics*, vol. 1, edited by H. Volland, pp. 111–178, CRC Press, Boca Raton, Fla.

Polk, C. (2000), In memoriam of Professor Herbert Koenig, *Bioelectromagn. Newsl.*, *156*, 8, May/June.

Polk, C., and F. Fitchen (1962), Schumann resonances of the Earth-ionsophere cavity—Extremely low frequency reception at Kingston, R.I., *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *66*, 313–318.

Rakov, V. A., and M. A. Uman (2005), *Lightning: Physics and Effects*, 687 pp., Cambridge Univ. Press, Cambridge, U. K.

Rösner, O. (1957), Die Ausbreitung elektromagnetischer TE-(H)-Wellen, erzeugt durch eine horizontale, stromdurchflossene Leiterschleife, in dem Hohlraum zwischen zwei konzentrischen Kugeln (Propagation of electromagnetic TE-(H)-waves, which are created by a horizontal, current carrying conducting loop, in the cavity between two concentric spheres), *Z. Angew. Phys.*, *9*, 448–453.

Rycroft, M. J. (1965), Resonances of the Earth-ionosphere cavity observed at Cambridge, England, *J. Res. Natl. Bur. Stand. U.S., Sect. D*, *69*, 1071–1081.

Rydbeck, O. E. H. (1944), On the propagation of radio waves, in *Trans. Chalmers Univ. Technol.*, no. 34, 1–169, Flanders Boktryckeri Aktiebolag, Goteborg, Sweden.

Rydbeck, O. E. H. (1948), On the forced electro-magnetic oscillations on spherical resonators, *Philos. Mag.*, *39*, 633–644.

Schelkunoff, S. A. (1943), *Electromagnetic Waves*, 530 pp., Van Nostrand Reinhold, Hoboken, N. J.

Schelkunoff, S. A. (1957), Sergei Alexander Schelkunoff, *IRE Trans. Microwave Theory Tech.*, *5*(3), 172.

Schlegel, K., and M. Füllekrug (2002), 50 Jahre Schumann-Resonanzen. Weltweite Ortung von Blitzen (50 years of Schumann resonances. Worldwide location finding of lightning), *Phys. Unserer Z.*, *33*(6), 256–261.

Schumann, W. O. (1952a), Über die strahlungslosen Eigenschwingungen einer leitenden Kugel, die von einer Luftschicht und einer Ionosphärenhülle umgeben ist (On the free eigen-oscillations of a conducting sphere, surrounded by an air film and an ionospheric shell), *Z. Naturforsch. A*, *7*, 149–154.

Schumann, W. O. (1952b), Über die Dämpfung der elektromagnetischen Eigenschwingungen des Systems Erde-Luft-Ionosphäre (On the damping of the electromagnetic eigen-oscillations of the Earth-air-ionosphere system), *Z. Naturforsch. A*, *7*, 250–252.

Schumann, W. O. (1952c), Über die Ausbreitung sehr langer elektrischer Wellen und der Blitzentladung um die Erde (On the propagation of very long electrical waves and the lightning discharge around the Earth), *Z. Angew. Phys.*, *4*, 474–480.

Schumann, W. O. (1952d), Über die Ausbreitung sehr langer elektrischer Wellen und das Wellenspektrum der Blitzentladung (On the propagation of very long electrical waves and the wave spectrum of a lightning discharge), *Naturwissenschaften*, *39*, 475–476.

Schumann, W. O. (1952e), Über die Ausbreitung sehr langer elektrischer Wellen um die Erde und die Signale des Blitzes (On the propagation of very long electrical waves around the Earth and the signals of the lightning discharge), *Nuovo Cimento*, *9*, 1116–1138.

Schumann, W. O. (1953), Über die Oberwellenfelder bei der Ausbreitung langer elektrischer Wellen um die Erde und die Signale des Blitzes (On the harmonic wave fields of the propagation of long electric waves round the Earth and the signals of a lightning stroke), *Naturwissenschaften*, *40*, 504–505.

Schumann, W. O. (1954a), Über die Oberfelder bei der Ausbreitung langer, elektrischer Wellen im System Erde-Luft-Ionosphäre und 2 Anwendungen (horizontaler und senkrechter Dipol) (On the harmonic fields of the propagation of long electric waves in the Earth-air-ionosphere system and two applications (horizontal and vertical dipole)), *Z. Angew. Phys.*, *6*, 35–43.

Schumann, W. O. (1954b), Über die Strahlung langer Wellen des horizontalen Dipols in dem Lufthohlraum zwischen Erde und Ionosphäre, I., (On the radiation of long waves from a horizontal dipole in the cavity between Earth and the ionosphere, I.), *Z. Angew. Phys.*, *6*, 225–229.

Schumann, W. O. (1954c), Über die Strahlung langer Wellen des horizontalen Dipols in dem Lufthohlraum zwischen Erde und Ionosphäre, I., (On the radiation of long waves from a horizontal dipole in the cavity between Earth and the ionosphere, II.), *Z. Angew. Phys.*, *6*, 267–271.

Schumann, W. O. (1954d), Über die Ausbreitung langer elektrischer Wellen um die Erde und einige Anwendungen auf Senderinterferenzen und Blitzsignale (On the propagation of long electric waves around the Earth and some applications to transmitter interferences and lightning signals), *Z. Angew. Phys.*, *6*, 346–352.

Schumann, W. O. (1955), Der Einfluß des Erdmagnetfeldes auf die Ausbreitung elektrischer Längstwellen (The influence of the Earth's magnetic field on the propagation of long electrical waves), *Z. Angew. Phys.*, *7*, 284–290.

Schumann, W. O. (1957), Über elektrische Eigenschwingungen des Hohlraumes Erde-Luft-Ionosphäre, erregt durch Blitzentladungen (On electrical eigen-oscillations of the Earth-air-ionosphere cavity, excited by lightning discharges), *Z. Angew. Phys.*, *9*, 373–378.

Schumann, W. O., and H. König (1954), Über die Beobachtung von "atmospherics" bei geringsten Frequenzen (On the observation of "atmospherics" at lowest frequencies), *Naturwissenschaften*, *41*, 183–184.

Sentman, D. D. (1995), Schumann resonances, in *Handbook of Atmospheric Electrodynamics*, vol. 1, pp. 267–295, edited by H. Volland, CRC Press, Boca Raton, Fla.

Shmoys, J. (1956), Long-range propagation of low-frequency radio waves between the earth and the ionosphere, *Proc. IRE*, *44*, 163–170.

Smith, E. K. (2000), James R. Wait—Remarkable scientist, *IEEE Trans. Antennas Propag.*, *48*(9), 1278–1286.

Stäblein, H.-G. (1955), Bemerkungen zur Ausbreitung langwelliger Blitzsignale um die Erde (Notes on the propagation of long wave lightning signals around the Earth), *Z. Angew. Phys.*, *7*, 290–291.

Stäblein, H.-G., and F. W. Heilbronner (1991), Winfried Otto Schumann, *IEEE Trans. Electr. Insul.*, *26*(3), 340–343.

Tesla, N. (1905), Art of transmitting electrical energy through the natural mediums, Patent 787.412, U.S. Patent and Trademark Off., Washington, D. C. (Available at http://freeinternetpress.com/mirrors/tesla_haarp/index.html.)

Thomson, J. J. (1893), *Notes on Recent Researches in Electricity and Magnetism, Intended as a Sequel to Professor Clerk-Maxwell's Treatise on Electricity and Magnetism)*, 578 pp., Clarendon, Oxford, U. K.

Thomson, W. (1872), Atmospheric electricity, in *Reprint of Papers on Electrostatics and Magnetism*, edited by W. T. Kelvin, pp. 192–236, Macmillan, New York.

Vladimirov, N. P., and N. G. Kleimenova (1962), On the structure of the Earth's natural electromagnetic field in the frequency range of 0.5–100 cps, *Bull. Acad. Sci. USSR Geophys. Ser.*, *6*(10), 852–855.

Volland, H. (1968), *Die Ausbreitung langer Wellen* (*Propagation of Long Waves*), 232 pp., Vieweg, Braunschweig, Germany.

Wait, J. R. (1960), On the theory of the slow-tail portion of atmospheric waveforms, *J. Geophys. Res.*, *65*(7), 1939–1946.

Wait, J. R. (1962), *Electromagnetic Waves in Stratified Media*, 372 pp., Pergamon, Oxford, U. K. (Reprinted by IEEE Press, Piscataway, N. J., 1996)

Wait, J. R. (1974a), Historical background and introduction to the special issue on extremely low frequency (ELF) communication, *IEEE Trans. Commun.*, *22*(4), 353–354.

Wait, J. R. (1974b), Dedication to Janis Galejs, *IEEE Trans. Commun.*, *22*(4), 355.

Wait, J. R. (1977), Propagation of ELF electromagnetic waves and project Sanguine/Seafarer, *IEEE J. Oceanic Eng.*, *2*(2), 161–172.

Watson, G. N. (1918), The transmission of electric waves round the Earth, *Proc. R. Soc., London, Ser. A*, *95*, 546–563.

Watson-Watt, R. A., J. F. Herd, and F. E. Lutkin (1937), On the nature of atmospherics—V, *Proc. R. Soc. London, Ser. A*, *162*, 267–285.

Watt, A. D., R. D. Crogham, and J. R. Wait (1967), Propagation, in *VLF Radio Engineering*, *Int. Ser. Monogr. Electromagn. Waves*, vol. 14, edited by A. D. Watt, pp. 171–394, Pergamon, Oxford, U. K.

Weidner, J. (1955), Über eine Verfeinerung der Theorie der Ausbreitung langer elektrischer Wellen um die Erde (On a refinement of the theory of long electrical waves propagation around the Earth), *Z. Angew. Phys.*, *7*, 77–82.

Whittaker, E. T. (1935), Hector Munro Macdonald 1865–1935, *Obituary Notices Fellows R. Soc. London*, *1*, 551–558.

Willis, H. F. (1948), Audio-frequency magnetic fluctuations, *Nature*, *161*, 887–888.

B. P. Besser, Institut für Weltraumforschung, Österreichische Akademie der Wissenschaften, Schmiedlstraße 6, A-8042 Graz, Austria. (bruno.besser@oeaw.ac.at)